HEAT AND MASS TRANSFER IN A SLIT GAP WITH SUBLIMATION OF A ROTATING DISK WALL

V. V. Faleev and N. V. Zavarzin

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A solution is obtained for the problem of the temperature distribution in a narrow slit gap between a motionless heated disk and a moving disk with a subliming coating. It is established that rotation leads to intensification of the heat-transfer process.

Analytical solution of the problem of the influence of sublimation on the heat transfer in the gap between motionless circular disks was considered in [1]. Distortion of the temperature profile across the slit in comparison with a plane channel was noted, as a consequence of the specific features of glow with cylindrical geometry. In the present work, the problem of [1] is generalized to the case of rotation of a disk with a subliming coating about a vertical symmetry axis.*

1. Consider steady laminar flow of solidifying-gas vapor in a narrow slit gap between circular, parallel, horizontal disks (Fig. 1). It is assumed that the lower disk is subjected to a constant uniformly distributed heat flux q. Sublimation of material at a constant rate v_c occurs from the upper disk, rotating uniformly at angular velocity ω about the vertical axis. The problem is solved in a cylindrical coordinate system, the z axis of which is directed along the disk axis and the r axis along the slit radius; the coordinate origin is at the center of the lower disk. Assuming that the problem is rotationally symmetric, the flow is described using the mass-transfer equations in the following dimensionless form:

$$u\frac{\partial u}{\partial r}+w\frac{\partial u}{\partial z}-\frac{v^2}{r}=-\frac{\partial p}{\partial r}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}+\frac{\partial^2 u}{\partial z^2}-\frac{u}{r^2}\right), \quad (1)$$

$$u\frac{\partial v}{\partial r}+w\frac{\partial v}{\partial z}+\frac{uv}{r}=\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 v}{\partial r^2}+\frac{1}{r}\frac{\partial v}{\partial r}+\frac{\partial^2 v}{\partial z^2}-\frac{v}{r^2}\right), \quad (2)$$

$$u\frac{\partial w}{\partial r}+w\frac{\partial w}{\partial z}=-\frac{\partial p}{\partial z}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}+\frac{\partial^2 w}{\partial z^2}\right), \quad (3)$$

$$\frac{\partial u}{\partial r}+\frac{u}{r}+\frac{\partial w}{\partial z}=0. \quad (4)$$

In reducing Eqs. (1)-(4) to dimensionless form, the components of the gas velocity vector were referred to v_c, the pressure to ρv_c^2, and the coordinates to the distance between the disks H.

The solution of Eqs. (1)-(4) is sought in the form

$$u=-\frac{r}{2}f'(z),\quad v=r\varphi(z),\quad w=f(z), \quad (5)$$

where f and φ are dimensionless functions of the axial coordinate z. In this case, the continuity equation — Eq. (4) — is identically satisfied, and the equations of motion — Eqs. (1)-(3) — take the form

$$f'''+\mathrm{Re}\left(\frac{1}{2}f'^2-ff''-2\varphi^2\right)=-\frac{2}{r}\mathrm{Re}\frac{\partial p}{\partial r}, \quad (6)$$

*The problem is solved in the quasisteady approximation: for a fixed height of the gas gap H, the temperature profile and gas velocity are regarded as steady.

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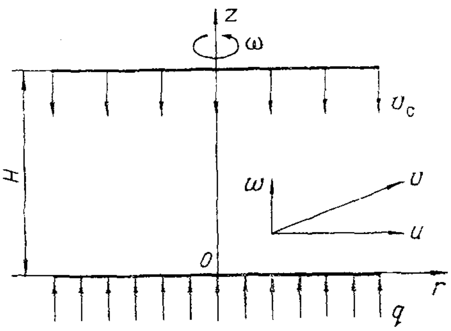

Fig. 1

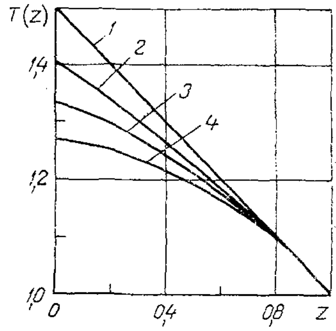

Fig. 2

Fig. 1. Flow pattern in slit gap.

Fig. 2. Temperature profile in gap between disks: 1) Pe = 0; 2, 3, 4) Pe = Re = m = 0.5; 2) ω = 0; 3) 20; 4) 28.3.

$$\varphi'' - \mathrm{Re}\,(f\varphi' - \varphi f') = 0, \tag{7}$$

$$f'' - ff'\,\mathrm{Re} = \mathrm{Re}\,\frac{\partial p}{\partial z}. \tag{8}$$

Eliminating the pressure in the gas p from Eqs. (6) and (8) by cross differentiation with respect to z and r, a system of two ordinary differential equations in the functions f and φ is obtained:

$$f''' + \mathrm{Re}\left(\frac{1}{2}f'^2 - ff'' - 2\varphi^2\right) = k = \mathrm{const}, \tag{9}$$

$$\varphi'' - \mathrm{Re}\,(f\varphi' - \varphi f') = 0. \tag{10}$$

The boundary conditions for Eqs. (9) and (10) are the adhesion conditions at the moving and motionless disks

$$\begin{aligned} &u = 0, \quad v = \omega r, \quad w = 1, \quad z = 1;\\ &u = 0, \quad v = 0, \quad w = 0, \quad z = 0. \end{aligned} \tag{11}$$

On the basis of Eq. (5), Eq. (11) is transformed to give

$$\begin{aligned} &f' = 0, \quad \varphi = \omega, \quad f = 1, \quad z = 1;\\ &f' = 0, \quad \varphi = 0, \quad f = 0, \quad z = 0. \end{aligned} \tag{12}$$

The system in Eqs. (9) and (10) is solved, taking account of Eq. (12), by the method of successive approximation, assuming that the Reynolds number of the flow is small. In this case, the equations and boundary conditions of the zero approximation are

$$f_0''' = k_0 = \mathrm{const}, \quad \varphi_0'' = 0, \tag{13}$$

$$\begin{aligned} &f_0' = 0, \quad \varphi_0 = \omega, \quad f_0 = 1, \quad z = 1,\\ &f_0' = 0, \quad \varphi_0 = 0, \quad f_0 = 0, \quad z = 0, \end{aligned} \tag{14}$$

and their solution is

$$f_0 = 3z^2 - 2z^3, \quad k_0 = -12, \quad \varphi_0 = \omega z. \tag{15}$$

The equation and boundary conditions for f in the next approximation are

$$f_1''' + \frac{1}{2}f_0'^2 - f_0 f_0'' - 2\varphi_0^2 = k_1 = \mathrm{const}, \tag{16}$$

$$f_1' = 0, \quad f_1 = 0, \quad z = 1; \quad f_1' = 0, \quad f_1 = 0, \quad z = 0. \tag{17}$$

Note that below, in solving the heat-conduction equation, only an expression for the components of the velocity vector w is required and, therefore, the first approximation for the function φ is not considered here.

Integration of Eq. (16), taking account of Eq. (17), gives

$$f_1 = \frac{1}{35} z^7 - \frac{1}{10} z^6 + \frac{9}{35} z^3 - \frac{13}{70} z^2 + \omega^2 \left(\frac{1}{30} z^5 - \frac{1}{10} z^3 + \frac{1}{15} z^2 \right). \quad (18)$$

Thus, with an accuracy of $o(\mathrm{Re}^2)$,

$$w = f(z) = 3z^2 - 2z^3 + \mathrm{Re} \left[\frac{1}{35} z^7 - \frac{1}{10} z^6 + \frac{9}{35} z^3 - \frac{13}{70} z^2 + \omega^2 \left(\frac{1}{30} z^5 - \frac{1}{10} z^3 + \frac{1}{15} z^2 \right) \right]. \quad (19)$$

When $\omega = 0$, Eq. (19) transforms to the well-known expression for w with sublimation in the gap between motionless disks [1].

2. Now consider the heat-conduction equation. Taking account of the assumption that the temperature difference along the slit radius is slight for the case of rotational symmetry, this equation may be written in the following dimensionless form:

$$w \,\mathrm{Pe} \frac{dT}{dz} = \frac{d^2T}{dz^2}. \quad (20)$$

Here and below, equations are brought to dimensionless form by referring the dimensional temperature to its value at the surface of the subliming disk T_C. The boundary conditions for Eq. (20) are the condition of constant sublimation temperature

$$T = 1, \quad z = 1 \quad (21)$$

and the heat-balance condition at the upper disk [1]

$$\frac{dT}{dz} = -m, \quad z = 1. \quad (22)$$

Then, the accurate solution of Eqs. (20)-(22) is written in the form

$$T(z) = 1 - m \int_1^z \exp\left[\mathrm{Pe} \int_1^z w(z)\, dz\right] dz. \quad (23)$$

Using Eq. (19), integration leads to the temperature distribution across the gap between the disks in the form

$$T(z) = 1 - m \exp\left[\left(\frac{1}{120} - \frac{\omega^2}{360}\right)\mathrm{Re} - \frac{1}{2}\right]\mathrm{Pe} \int_1^z \exp\left\{z^3 - \frac{1}{2} z^4 + \right.$$
$$\left. + \mathrm{Re}\left[\frac{z^8}{280} - \frac{z^7}{70} + \frac{9z^4}{140} - \frac{13z^3}{210} + \omega^2\left(\frac{z^6}{180} - \frac{z^4}{40} + \frac{z^3}{45}\right)\right]\right\} \mathrm{Pe}\, dz. \quad (24)$$

Equation (24) is fairly unwieldy and inconvenient for use in specific numerical calculations. It may be simplified by assuming that, as well as the Reynolds number, the Peclet number is also sufficiently small. Neglecting terms including the product PeRe (terms with ω^2 are retained in view of the possible large values of ω), the following simpler relation is obtained

$$T(z) = 1 - m \exp\left[\left(-\frac{\omega^2}{360}\mathrm{Re} - \frac{1}{2}\right)\mathrm{Pe}\right] \int_1^z \exp\left\{\left[z^3 - \frac{z^4}{2} + \mathrm{Re}\left(\frac{z^6}{180} - \frac{z^4}{40} + \frac{z^3}{45}\right)\omega^2\right]\mathrm{Pe}\right\} dz. \quad (25)$$

Using the approximation of an exponential with an infinitesimal exponent, integration leads to the final form of the dependence of the temperature on the coordinate across the gap

$$T(z) = 1 - m\left[z - 1 - \left(\frac{z^5}{10} - \frac{z^4}{4} + \frac{z}{2} - \frac{7}{20}\right)\mathrm{Pe} + \left(\frac{z^7}{1260} - \frac{z^5}{200} + \frac{z^4}{180} - \frac{z}{360} + \frac{1}{700}\right)\mathrm{Re}\,\mathrm{Pe}\,\omega^2\right]. \quad (26)$$

It is simple to establish, by direct substitution, that, despite the various simplifications, the expression for T(z) in Eq. (26) absolutely accurately satisfies the boundary conditions in Eqs. (21) and (22) and satisfies Eq. (20) with an accuracy of $o(\mathrm{Pe}^2, \mathrm{PeRe}, \mathrm{Re}^2)$.

The form of the temperature profile in the gap between the disks according to Eq. (26) is shown in Fig. 2 for various values of the angular velocity of rotation of the upper disk ω. It is evident that, with increase in ω, the temperature of the heated disk ($z = 0$) falls. This indicates higher efficiency of the thermal protection in the case of a rotating disk with a subliming coating.

Note, in conclusion, that in a series of cases it is more expedient to estimate the heat-transfer intensity not from the change in temperature profile in Eq. (26) but from the value of the dimensionless heat-transfer coefficient Nu at the channel walls

$$\mathrm{Nu} = \frac{1}{T(1) - T(0)} \frac{dT}{dz}. \tag{27}$$

Here the temperature gradient dT/dz is determined in the following form, taking account of the above simplifying assumptions

$$\frac{dT}{dz} = -m\left[1 + \left(z^3 - \frac{z^4}{2} - \frac{1}{2}\right)\mathrm{Pe} + \left(\frac{z^6}{180} - \frac{z^4}{40} + \frac{z^3}{45} - \frac{1}{360}\right)\mathrm{Re\,Pe}\,\omega^2\right]. \tag{28}$$

In this case,

$$\mathrm{Nu}(1) = \frac{1}{1 - \frac{7}{20}\mathrm{Pe} - \frac{1}{700}\mathrm{Re\,Pe}\,\omega^2}, \quad \mathrm{Nu}(0) = \left(1 - \frac{1}{2}\mathrm{Pe} - \frac{1}{360}\mathrm{Re\,Pe}\,\omega^2\right)\mathrm{Nu}(1). \tag{29}$$

It is readily evident that Nu(1) increases with increase in ω, while Nu(0) decreases. At the same time, the temperature difference across the slit

$$\Delta T = T(0) - T(1) = \frac{m}{\mathrm{Nu}(1)}$$

decreases in inverse proportion to Nu(1) as ω increases, which indicates an overall increase in heat-transfer intensity on account of the rotation of the subliming disk.

NOTATION

H, height of the gas gap; q, heat-flux intensity; ω, angular velocity of rotation; v_c, sublimation rate; z, r, coordinates of cylindrical system; ρ, density; p, pressure in gas gap; T_c, sublimation temperature; Nu, Re, Pe, Nusselt, Reynolds, and Peclet numbers; r_c, heat of sublimation; $m = \mathrm{Pe} r_c / T_c c_p$, dimensionless complex.

LITERATURE CITED

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